

Space-time compactification/decompactification transitions via lightlike branes

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Abstract We consider Einstein-Maxwell-Kalb-Ramond gravity-matter system in bulk space-time interacting self-consistently with two (widely separated) codimension-one electrically charged *lightlike* branes. The lightlike brane dynamics is explicitly given by manifestly reparametrization invariant world-volume actions in two equivalent dual to each other formulations (Polyakov-type and Nambu-Goto-type ones) proposed in our previous work. We find an explicit solution of the pertinent Einstein-Maxwell-Kalb-Ramond-lightlike-brane equations of motion describing a “two-throat” wormhole-like space-time consisting of a “left” compactified Bertotti-Robinson universe connected to a “middle” non-compact Reissner-Nordström-de-Sitter space-time region, which in turn is connected to another “right” compactified Bertotti-Robinson universe. Each of the lightlike branes automatically occupies one of the “throats”, so that they dynamically induce a sequence of spontaneous space-time compactification/decompactification transitions.

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1 Introduction

Lightlike branes (*LL-branes* for short) play an increasingly significant role in general relativity and modern non-perturbative string theory. Mathematically they represent singular null hypersurfaces in Riemannian space-time which provide dynamical description of various physically important cosmological and astrophysical phenomena such as: (1) impulsive lightlike signals arising in cataclysmic astrophysical events (supernovae, neutron star collisions) [1]; (2) dynamics of horizons in black hole physics—the so called “membrane paradigm” [2]; (3) the thin-wall approach to domain walls coupled to gravity [3–6].

More recently, the relevance of *LL-branes* in the context of non-perturbative string theory has also been recognized, specifically, as the so called *H-branes* describing quantum horizons (black hole and cosmological) [7], as Penrose limits of baryonic *D-branes* [8], etc (see also Refs. [9–11]).

A characteristic feature of the formalism for *LL-branes* in the pioneering papers [3–6] in the context of gravity and cosmology is that they have been exclusively treated in a “phenomenological” manner, i.e., without specifying an underlying Lagrangian dynamics from which they may originate. As a partial exception, in a more recent paper [12] brane actions in terms of their pertinent extrinsic geometry have been proposed which generically describe non-lightlike branes, whereas the lightlike branes are treated as a limiting case.

On the other hand, in the last few years we have proposed in a series of papers [13–21] a new class of concise manifestly reparametrization invariant world-volume Lagrangian actions, providing a derivation from first principles of the *LL-brane* dynamics. The following characteristic features of the new *LL-branes* drastically distinguish them from ordinary Nambu-Goto branes:

- (a) They describe intrinsically lightlike modes, whereas Nambu-Goto branes describe massive ones.
- (b) The tension of the *LL-brane* arises as an *additional dynamical degree of freedom*, whereas Nambu-Goto brane tension is a given ad hoc constant. The latter characteristic feature significantly distinguishes our *LL-brane* models from the previously proposed *tensionless p-branes* (for a review, see Ref. [22]). The latter rather resemble *p-dimensional* continuous distributions of independent massless point-particles without cohesion among the latter.
- (c) Consistency of *LL-brane* dynamics in a spherically or axially symmetric gravitational background of codimension one requires the presence of an event horizon which is automatically occupied by the *LL-brane* (“horizon straddling” according to the terminology of Ref. [5]).
- (d) When the *LL-brane* moves as a *test* brane in spherically or axially symmetric gravitational backgrounds its dynamical tension exhibits exponential “infla-

tion/deflation” time behavior [17, 18]—an effect similar to the “mass inflation” effect around black hole horizons [23, 24].

An intriguing novel application of *LL-branes* as natural self-consistent gravitational sources for *wormhole* space-times has been developed in a series of recent papers [19–21, 25, 26].

Before proceeding let us recall that the concept of “wormhole space-time” was born in the classic work of Einstein and Rosen [27], where they considered matching along the horizon of two identical copies of the exterior Schwarzschild space-time region (subsequently called *Einstein-Rosen “bridge”*). Another corner stone in wormhole physics is the seminal work of Morris and Thorne [28, 29], who studied for the first time *traversable Lorentzian wormholes*.

In what follows, when discussing wormholes we will have in mind the physically important class of “thin-shell” traversable Lorentzian wormholes first introduced by Visser [30–32]. For a comprehensive review of wormhole space-times, see Refs. [32–35].

In our earlier work [19–21, 25, 26] we have constructed various types of wormhole solutions in self-consistent systems of bulk gravity and bulk gauge fields (Maxwell and Kalb-Ramond) coupled to *LL-branes* where the latter provide the appropriate stress energy tensors, electric currents and dynamically generated space-varying cosmological constant terms consistently derived from well-defined world-volume *LL-brane* Lagrangian actions.

The original Einstein-Rosen “bridge” manifold [27] appears as a particular case of the construction of spherically symmetric wormholes produced by *LL-branes* as gravitational sources occupying the wormhole throats (Refs. [21, 25]). Thus, we are lead to the important conclusion that consistency of Einstein equations of motion yielding the original Einstein-Rosen “bridge” as well-defined solution necessarily requires the presence of *LL-brane* energy-momentum tensor as a source on the right hand side¹.

More complicated examples of spherically and axially symmetric wormholes with Reissner-Nordström and rotating cylindrical geometry, respectively, have been

¹ The crucial role of the presence of the *LL-brane* gravitational source producing the Einstein-Rosen “bridge” was not recognized in the original classic paper [27]. On the other hand, the failure of the original Einstein-Rosen “bridge” metric to satisfy the vacuum Einstein equations at the matching hypersurface—the Schwarzschild horizon, which now serves as wormhole “throat”, has been noticed in Ref. [27], where in (3a) the authors multiply Ricci tensor by an appropriate power of the determinant of the “bridge” metric vanishing at the “throat” so as to enforce fulfillment of the vacuum Einstein equations everywhere, including at the “throat”. To avoid confusion, let us particularly emphasize that here we consider the Einstein-Rosen “bridge” in its original formulation in Ref. [27] as a four-dimensional space-time manifold consisting of two copies of the exterior Schwarzschild space-time region matched along the horizon. On the other hand, the nomenclature of “Einstein-Rosen bridge” in several standard textbooks (e.g. Ref. [36]) uses the Kruskal-Szekeres manifold. The latter notion of “Einstein-Rosen bridge” is not equivalent to the original construction in Ref. [27]. Namely, the two regions in Kruskal-Szekeres space-time corresponding to the outer Schwarzschild space-time region ($r > 2m$) and labeled (*I*) and (*III*) in Ref. [36] are generally *disconnected* and share only a two-sphere (the angular part) as a common border ($U = 0, V = 0$ in Kruskal-Szekeres coordinates), whereas in the original Einstein-Rosen “bridge” construction the boundary between the two identical copies of the outer Schwarzschild space-time region ($r > 2m$) is a three-dimensional hypersurface ($r = 2m$).

explicitly constructed via *LL-branes* in Refs. [19–21]. Namely, two copies of the exterior space-time region (i.e., the space-time region beyond the respective outer event horizon) of a Reissner–Nordström or rotating cylindrical black hole, respectively, are matched via *LL-brane* along what used to be the outer horizon of the respective full black hole space-time manifold. In this way one obtains a wormhole solution which combines the features of the Einstein–Rosen “bridge” on the one hand (with wormhole throat at horizon), and the features of Misner–Wheeler wormholes [37], i.e., exhibiting the so called “charge without charge” phenomenon², on the other hand.

The results of Refs. [19–21] have been further extended in [26,38] to the case of *asymmetric* wormholes, describing two “universes” with different (spherically symmetric) geometries of black hole type connected via a “throat” materialized by the pertinent gravitational source—an electrically charged *LL-brane*, sitting on their common horizon. In [26] it has been shown that as a result of the well-defined world-volume dynamics of the *LL-brane* coupled self-consistently to gravity and bulk space-time gauge fields, it creates a “left universe” comprising the exterior Schwarzschild-de-Sitter space-time region beyond the Schwarzschild horizon and where the cosmological constant is dynamically generated, and a “right universe” comprising the exterior Reissner–Nordström region beyond the outer Reissner–Nordström horizon with dynamically generated Coulomb field-strength. Both “universes” are glued together by the *LL-brane* occupying their common horizon. Similarly, the *LL-brane* can dynamically generate a non-zero cosmological constant in the “right universe”, in which case it connects a purely Schwarzschild “left universe” with a Reissner–Nordström-de-Sitter “right universe”. Furthermore, in Ref. [38] another new type of wormhole solution to Einstein–Maxwell equations has been constructed, which describes a “right universe” comprising the exterior Reissner–Nordström space-time region beyond the outer Reissner–Nordström horizon, connected through a “throat” materialized by a *LL-brane* with a “left universe” being a Bertotti–Robinson space-time with two compactified spatial dimensions [39,40]³. Thus, the *LL-brane* has been shown to dynamically induce space-time compactification.

Let us note that previously the junction of a compactified space-time (of Bertotti–Robinson type) to an uncompactified space-time through a wormhole has been studied in a different setting using *timelike* matter on the junction hypersurface [43]. Recently [44] a general class of solutions of Einstein–Maxwell–Dilaton system have been found describing an interpolation between Reissner–Nordström and Bertotti–Robinson spacetimes. Also, in a different context a string-like (flux tube) object with similar features to Bertotti–Robinson solution has been constructed [45] which interpolates between uncompactified space-time regions.

In the present paper we will further broaden the application of *LL-branes* in the context of wormhole physics by constructing a “two-throat” wormhole-type solution

² Misner and Wheeler [37] realized that wormholes connecting two asymptotically flat space times provide the possibility of “charge without charge”, i.e., electromagnetically non-trivial solutions where the lines of force of the electric field flow from one universe to the other without a source and giving the impression of being positively charged in one universe and negatively charged in the other universe.

³ Originally space-time solution with two compactified spatial dimensions was found by Levi-Civita [41]; see also [42].

to Einstein-Maxwell system self-consistently interacting with *two widely separated* electrically charged *LL-branes* describing a sequence of a decompactification and a subsequent compactification transitions interpolating between two Bertotti-Robinson universes with different sizes of the compactified dimensions, i.e., a sort of space-time compactification “kink” solution.

The presentation of the material in the paper is as follows. Sections 2 and 3 contain the basics of our formalism. In Sect. 2 we review the reparametrization-invariant world-volume Lagrangian formulation of *LL-branes* in both the Polyakov-type and Nambu-Goto-type forms. In Sect. 3 we briefly describe the main properties of *LL-brane* dynamics in spherically symmetric gravitational backgrounds stressing particularly on the “horizon straddling” phenomenon and the dynamical cosmological constant generation. Section 4 presents the systematic Lagrangian formulation of bulk Einstein-Maxwell-Kalb-Ramond system self-consistently interacting with two distantly separated charged codimension-one *LL-branes*. Section 5 contains our principal result—the explicit construction of a new asymmetric “two-throat” wormhole solution of the above coupled gravity/gauge-*LL-brane* system describing:

- (i) Bertotti-Robinson compactified “left universe” connected via the first *LL-brane* to the
- (ii) uncompactified “middle universe” comprising the intermediate space-time region of Reissner-Nordström-de-Sitter universe between the middle (outer Reissner-Nordström) horizon and the outmost (de Sitter) horizon, connected in turn via the second *LL-brane* to
- (iii) another Bertotti-Robinson “right universe” with *different* (w.r.t. (i)) size of the compactified dimensions.

In the concluding Sect. 6 we briefly discuss the issue of traversability of the above wormhole solution as well as possible generalizations to higher space-time dimensions.

2 World-volume formulation of lightlike brane dynamics

2.1 Polyakov-type formulation

There exist two equivalent *dual to each other* manifestly reparametrization invariant world-volume Lagrangian formulations of *LL-branes* [13–21, 25, 46]. First, let us consider the Polyakov-type formulation where the *LL-brane* world-volume action is given as:

$$S_{\text{Pol}} = \int d^{p+1} \sigma \Phi \left[-\frac{1}{2} \gamma^{ab} g_{ab} + L(F^2) \right]. \quad (1)$$

Here the following notions and notations are used:

- (a) Φ is an alternative non-Riemannian integration measure density (volume form) on the p -brane world-volume manifold:

$$\Phi \equiv \frac{1}{(p + 1)!} \varepsilon^{a_1 \dots a_{p+1}} H_{a_1 \dots a_{p+1}}(B), \quad H_{a_1 \dots a_{p+1}}(B) = (p + 1) \partial_{[a_1} B_{a_2 \dots a_{p+1}]}, \tag{2}$$

instead of the usual $\sqrt{-\gamma}$. Here $\varepsilon^{a_1 \dots a_{p+1}}$ is the alternating symbol ($\varepsilon^{01 \dots p} = 1$), γ_{ab} ($a, b = 0, 1, \dots, p$) indicates the intrinsic Riemannian metric on the world-volume, and $\gamma = \det \|\gamma_{ab}\|$. $H_{a_1 \dots a_{p+1}}(B)$ denotes the field-strength of an auxiliary world-volume antisymmetric tensor gauge field $B_{a_1 \dots a_p}$ of rank p . As a special case one can build $H_{a_1 \dots a_{p+1}}$ in terms of $p + 1$ auxiliary world-volume scalar fields $\{\varphi^I\}_{I=1}^{p+1}$:

$$H_{a_1 \dots a_{p+1}} = \varepsilon_{I_1 \dots I_{p+1}} \partial_{a_1} \varphi^{I_1} \dots \partial_{a_{p+1}} \varphi^{I_{p+1}}. \tag{3}$$

Note that γ_{ab} is independent of the auxiliary world-volume fields $B_{a_1 \dots a_p}$ or φ^I . The alternative non-Riemannian volume form (2) has been first introduced in the context of modified standard (non-lightlike) string and p -brane models in Refs. [47, 48].

- (b) $X^\mu(\sigma)$ are the p -brane embedding coordinates in the bulk D -dimensional space time with bulk Riemannian metric $G_{\mu\nu}(X)$ with $\mu, \nu = 0, 1, \dots, D - 1$; $(\sigma) \equiv (\sigma^0 \equiv \tau, \sigma^i)$ with $i = 1, \dots, p$; $\partial_a \equiv \frac{\partial}{\partial \sigma^a}$.
- (c) g_{ab} is the induced metric on world-volume:

$$g_{ab} \equiv \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X), \tag{4}$$

which becomes singular on-shell (manifestation of the lightlike nature, cf. second (12) below).

- (d) $L(F^2)$ is the Lagrangian density of another auxiliary $(p - 1)$ -rank antisymmetric tensor gauge field $A_{a_1 \dots a_{p-1}}$ on the world-volume with p -rank field-strength and its dual:

$$F_{a_1 \dots a_p} = p \partial_{[a_1} A_{a_2 \dots a_p]}, \quad F^{*a} = \frac{1}{p!} \frac{\varepsilon^{a a_1 \dots a_p}}{\sqrt{-\gamma}} F_{a_1 \dots a_p}. \tag{5}$$

$L(F^2)$ is arbitrary function of F^2 with the short-hand notation:

$$F^2 \equiv F_{a_1 \dots a_p} F_{b_1 \dots b_p} \gamma^{a_1 b_1} \dots \gamma^{a_p b_p}. \tag{6}$$

Rewriting the action (1) in the following equivalent form:

$$S_{\text{Pol}} = - \int d^{p+1} \sigma \chi \sqrt{-\gamma} \left[\frac{1}{2} \gamma^{ab} g_{ab} - L(F^2) \right], \quad \chi \equiv \frac{\Phi}{\sqrt{-\gamma}} \tag{7}$$

with Φ the same as in (2), we find that the composite field χ plays the role of a dynamical (variable) brane tension.⁴

Let us now consider the equations of motion corresponding to (1) w.r.t. $B_{a_1\dots a_p}$:

$$\partial_a \left[\frac{1}{2} \gamma^{cd} g_{cd} - L(F^2) \right] = 0 \quad \longrightarrow \quad \frac{1}{2} \gamma^{cd} g_{cd} - L(F^2) = M, \tag{8}$$

where M is an arbitrary integration constant. The equations of motion w.r.t. γ^{ab} read:

$$g_{ab} - 2F^2 L'(F^2) \left[\gamma_{ab} - \frac{F_a^* F_b^*}{F^{*2}} \right] = 0, \tag{9}$$

where F^{*a} is the dual field strength (5). Equation (9) can be viewed as p -brane analogues of the string Virasoro constraints.

Remark 1 Before proceeding, let us mention that both the auxiliary world-volume field $B_{a_1\dots a_p}$ entering the non-Riemannian integration measure density (2), as well as the intrinsic world-volume metric γ_{ab} are *non-dynamical* degrees of freedom in the action (1), or equivalently, in (7). Indeed, there are no (time-)derivatives w.r.t. γ_{ab} , whereas the action (1) (or (7)) is *linear* w.r.t. the velocities $\partial_0 B_{a_1\dots a_p}$. Thus, (1) is a constrained dynamical system, i.e., a system with gauge symmetries including the gauge symmetry under world-volume reparametrizations, and both (8)–(9) are in fact *non-dynamical constraint* equations (no second-order time derivatives present). Their meaning as constraint equations is best understood within the framework of the Hamiltonian formalism for the action (1). The latter can be developed in strict analogy with the Hamiltonian formalism for a simpler class of modified *non-lightlike* p -brane models based on the alternative non-Riemannian integration measure density (2), which was previously proposed in Ref. [52] (for details, we refer to Sects. 2 and 3 of Ref. [52]). In particular, (9) can be viewed as p -brane analogues of the string Virasoro constraints.

Taking the trace in (9) and comparing with (8) implies the following crucial relation for the Lagrangian function $L(F^2)$:

$$L(F^2) - pF^2 L'(F^2) + M = 0 \quad \longrightarrow \quad F^2 = F^2(M) = \text{const}, \tag{10}$$

which determines F^2 (6) on-shell as certain function of the integration constant M (8):

$$F^2 = F^2(M) = \text{const}. \tag{11}$$

Here and below $L'(F^2)$ denotes derivative of $L(F^2)$ w.r.t. the argument F^2 .

⁴ The notion of dynamical brane tension has previously appeared in different contexts in Refs. [49–51].

The next and most profound consequence of (9) is that the induced metric (4) on the world-volume of the p -brane model (1) is *singular* on-shell (as opposed to the induced metric in the case of ordinary Nambu-Goto branes):

$$g_{ab}F^{*b} \equiv \partial_a X^\mu G_{\mu\nu} \left(\partial_b X^\nu F^{*b} \right) = 0. \tag{12}$$

Equation (12) is the manifestation of the *lightlike* nature of the p -brane model (1) (or (7)), namely, the tangent vector to the world-volume $F^{*a} \partial_a X^\mu$ is *lightlike* w.r.t. metric of the embedding space-time.

Further, the equations of motion w.r.t. world-volume gauge field $A_{a_1 \dots a_{p-1}}$ (with χ as defined in (7) and accounting for the constraint (11)) read:

$$\partial_{[a} (F_{b]}^* \chi) = 0. \tag{13}$$

Finally, the X^μ equations of motion produced by the (1) read:

$$\partial_a \left(\chi \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) + \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu = 0 \tag{14}$$

where $\Gamma_{\nu\lambda}^\mu = \frac{1}{2} G^{\mu\kappa} (\partial_\nu G_{\kappa\lambda} + \partial_\lambda G_{\kappa\nu} - \partial_\kappa G_{\nu\lambda})$ is the Christoffel connection for the external metric.

2.2 Nambu-Goto-type formulation

Equation (13) allows us to introduce the dual “gauge” potential u (dual w.r.t. world-volume gauge field $A_{a_1 \dots a_{p-1}}$ (5)):

$$F_a^* = c_p \frac{1}{\chi} \partial_a u, \quad c_p = \text{const.} \tag{15}$$

Relation (15) enables us to rewrite (9) (the lightlike constraint) in terms of the dual potential u in the form:

$$\gamma_{ab} = \frac{1}{b_0} g_{ab} - \frac{b_0^{p-2}}{\chi^2} \partial_a u \partial_b u \quad \text{with} \quad b_0 \equiv 2F^2 L' \left(F^2 \right) \Big|_{F^2 = F^2(M)} = \text{const.} \tag{16}$$

From (15) we obtain the relation:

$$\chi^2 = -b_0^{p-2} \gamma^{ab} \partial_a u \partial_b u, \tag{17}$$

and the Bianchi identity $\nabla_a F^{*a} = 0$ becomes:

$$\partial_a \left(\frac{1}{\chi} \sqrt{-\gamma} \gamma^{ab} \partial_b u \right) = 0. \tag{18}$$

It is straightforward to prove that the system of (14), (18) and (17) for (X^μ, u, χ) , which are equivalent to the equations of motion (8–13), (14) resulting from the original Polyakov-type *LL-brane* action (1), can be equivalently derived from the following *dual* Nambu-Goto-type world-volume action:

$$S_{\text{NG}} = - \int d^{p+1} \sigma T \sqrt{\left| \det \|g_{ab} - \epsilon \frac{1}{T^2} \partial_a u \partial_b u\| \right|}, \quad \epsilon = \pm 1. \tag{19}$$

Here again g_{ab} indicates the induced metric on the world-volume (4) and T is dynamical variable tension simply proportional to χ (see (24) below). The choice of the sign in (19) does not have physical effect because of the non-dynamical nature of the u -field.

The corresponding equations of motion w.r.t. X^μ, u and T read accordingly:

$$\partial_a \left(T \sqrt{|\tilde{g}|} \tilde{g}^{ab} \partial_b X^\mu \right) + T \sqrt{|\tilde{g}|} \tilde{g}^{ab} \partial_a X^\lambda \partial_b X^\nu \Gamma_{\lambda\nu}^\mu = 0, \tag{20}$$

$$\partial_a \left(\frac{1}{T} \sqrt{|\tilde{g}|} \tilde{g}^{ab} \partial_b u \right) = 0, \quad T^2 + \epsilon \tilde{g}^{ab} \partial_a u \partial_b u = 0, \tag{21}$$

where we have introduced the convenient notations:

$$\tilde{g}_{ab} = g_{ab} - \epsilon \frac{1}{T^2} \partial_a u \partial_b u, \quad \tilde{g} \equiv \det \| \tilde{g}_{ab} \|, \tag{22}$$

and \tilde{g}^{ab} is the inverse matrix w.r.t. \tilde{g}_{ab} .

From the definition (22) and second (21) one easily finds that the induced metric on the world-volume is singular on-shell (cf. (12) above):

$$g_{ab} \left(\tilde{g}^{bc} \partial_c u \right) = 0 \tag{23}$$

exhibiting the lightlike nature of the p -brane described by (19). Also, comparing (22) with (16) and assuming $\epsilon = 1$ yields relations with the world-volume metric and the dynamical tension from the Polyakov-type formulation:

$$\gamma_{ab} = \frac{1}{b_0} \tilde{g}_{ab}, \quad \chi^2 = b_0^{p-1} T^2, \tag{24}$$

with b_0 as in (16). Yet the meaning of the constant b_0 , which appears as an integration constant within the Polyakov-type formulation (cf. (16)), is different within the Nambu-Goto-type formulation (19) where it arises as an arbitrary gauge-fixing parameter for the world-volume reparametrization invariance—see (40) below.

Remark 2 Similarly to the ordinary bosonic p -brane we can rewrite the Nambu-Goto-type action for the *LL-brane* (19) in a Polyakov-like form by employing an

intrinsic Riemannian world-volume metric γ_{ab} as in (1):

$$S_{\text{NG-Pol}} = -\frac{1}{2} \int d^{p+1} \sigma T \sqrt{-\gamma} \left[\gamma^{ab} \left(g_{ab} - \epsilon \frac{1}{T^2} \partial_a u \partial_b u \right) - \epsilon b_0 (p-1) \right] \tag{25}$$

which exhibits more explicitly the duality w.r.t. (1).

Remark 3 Let us note that duality between the Polyakov-type (1) and Nambu-Goto-type (19) formulations of *LL-brane* dynamic strictly exists provided we choose the sign $\epsilon = 1$ in the Nambu-Goto-type action (19). In what follows we will use the Nambu-Goto-type formulation to describe one of the *LL-branes* self-consistently coupled to the bulk gravity/gauge-field system where consistency of the pertinent solution requires $\epsilon = -1$.

2.3 Coupling to bulk space-time gauge fields

Using the above world-volume Lagrangian framework one can add in a natural way [13–20] couplings of the *LL-brane* to bulk space-time Maxwell \mathcal{A}_μ and Kalb-Ramond $\mathcal{A}_{\mu_1 \dots \mu_{D-1}}$ gauge fields (the latter—in the case of codimension one *LL-branes*, i.e., for $D = (p+1) + 1$). In the Polyakov-type formalism we have:

$$\begin{aligned} \tilde{S}_{\text{Pol}}[q, \beta] = & S_{\text{Pol}} - q \int d^{p+1} \sigma \varepsilon^{ab_1 \dots b_p} F_{b_1 \dots b_p} \partial_a X^\mu \mathcal{A}_\mu \\ & - \frac{\beta}{(p+1)!} \int d^{p+1} \sigma \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} \mathcal{A}_{\mu_1 \dots \mu_{p+1}} \end{aligned} \tag{26}$$

with S_{LL} as in (1). The *LL-brane* constraint (8)–(9) are not affected by the bulk space-time gauge field couplings, whereas (13)–(14) acquire the form:

$$\partial_{[a} \left(F_{b]}^* \chi L'(F^2) \right) + \frac{q}{4} \partial_a X^\mu \partial_b X^\nu \mathcal{F}_{\mu\nu} = 0; \tag{27}$$

$$\begin{aligned} \partial_a \left(\chi \sqrt{-\gamma} \gamma^{ab} \partial_b X^\mu \right) + \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\nu \partial_b X^\lambda \Gamma_{\nu\lambda}^\mu - q \varepsilon^{ab_1 \dots b_p} F_{b_1 \dots b_p} \partial_a X^\nu \mathcal{F}_{\lambda\nu} G^{\lambda\mu} \\ - \frac{\beta}{(p+1)!} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} \mathcal{F}_{\lambda\mu_1 \dots \mu_{p+1}} G^{\lambda\mu} = 0. \end{aligned} \tag{28}$$

Here χ is the dynamical brane tension as in (7), and

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu, \quad \mathcal{F}_{\mu_1 \dots \mu_D} = D \partial_{[\mu_1} \mathcal{A}_{\mu_2 \dots \mu_D]} = \mathcal{F} \sqrt{-G} \varepsilon_{\mu_1 \dots \mu_D} \tag{29}$$

are the field-strengths of the electromagnetic \mathcal{A}_μ and Kalb-Ramond $\mathcal{A}_{\mu_1 \dots \mu_{D-1}}$ gauge potentials [53,54].

The dual counterpart of the action (26) within the Nambu-Goto-type formalism reads:

$$\begin{aligned} \tilde{S}_{\text{NG}}[\bar{q}, \bar{\beta}] = & - \int d^{p+1} \sigma T \sqrt{\left| \det \left\| g_{ab} - \epsilon \frac{1}{T^2} (\partial_a u + \bar{q} \mathcal{A}_a)(\partial_b u + \bar{q} \mathcal{A}_b) \right\| \right|} \\ & - \frac{\bar{\beta}}{(p+1)!} \int d^{p+1} \sigma \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} \mathcal{A}_{\mu_1 \dots \mu_{p+1}} \end{aligned} \quad (30)$$

with g_{ab} denoting the induced metric on the world-volume (4) and $\mathcal{A}_a \equiv \partial_a X^\mu \mathcal{A}_\mu$. Accordingly, using the short-hand notation generalizing (22):

$$\bar{g}_{ab} \equiv g_{ab} - \epsilon \frac{1}{T^2} (\partial_a u + \bar{q} \mathcal{A}_a)(\partial_b u + \bar{q} \mathcal{A}_b), \quad \mathcal{A}_a \equiv \partial_a X^\mu \mathcal{A}_\mu, \quad (31)$$

the equations of motion w.r.t. X^μ , u and T acquire the form:

$$\begin{aligned} \partial_a \left(T \sqrt{|\bar{g}|} \bar{g}^{ab} \partial_b X^\mu \right) + T \sqrt{|\bar{g}|} \bar{g}^{ab} \partial_a X^\lambda \partial_b X^\nu \Gamma_{\lambda\nu}^\mu \\ + \epsilon \frac{\bar{q}}{T} \sqrt{|\bar{g}|} \bar{g}^{ab} \partial_a X^\nu (\partial_b u + \bar{q} \mathcal{A}_b) \mathcal{F}^{\lambda\nu} G^{\mu\lambda} \\ - \frac{\bar{\beta}}{(p+1)!} \varepsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} \mathcal{F}_{\lambda\mu_1 \dots \mu_{p+1}} G^{\lambda\mu} = 0, \end{aligned} \quad (32)$$

$$\partial_a \left(\frac{1}{T} \sqrt{|\bar{g}|} \bar{g}^{ab} (\partial_b u + \bar{q} \mathcal{A}_b) \right) = 0, \quad (33)$$

$$T^2 + \epsilon \bar{g}^{ab} (\partial_a u + \bar{q} \mathcal{A}_a)(\partial_b u + \bar{q} \mathcal{A}_b) = 0. \quad (34)$$

The on-shell singularity of the induced metric g_{ab} (4), i.e., the lightlike property, now reads (cf. (23)):

$$g_{ab} \left(\bar{g}^{bc} (\partial_c u + \bar{q} \mathcal{A}_c) \right) = 0. \quad (35)$$

3 Lightlike brane dynamics in various types of gravitational backgrounds

3.1 Gauge-fixed lightlike brane equations of motion

Going back to the Polyakov-type formalism (Sect. 2.1), world-volume reparametrization invariance allows us to introduce the standard synchronous gauge-fixing conditions:

$$\gamma^{0i} = 0 \quad (i = 1, \dots, p), \quad \gamma^{00} = -1. \quad (36)$$

Also, we will use a natural ansatz for the “electric” part of the auxiliary world-volume gauge field-strength (5):

$$F^{*i} = 0 \quad (i = 1, \dots, p), \quad \text{i.e.} \quad F_{0i_1 \dots i_{p-1}} = 0, \quad (37)$$

meaning that we choose the lightlike direction in (12) to coincide with the brane proper-time direction on the world-volume ($F^{*a} \partial_a \sim \partial_\tau$). The Bianchi identity ($\nabla_a F^{*a} = 0$) together with (36)–(37) and the definition for the dual field-strength in (5) imply:

$$\partial_\tau \gamma^{(p)} = 0 \quad \text{where} \quad \gamma^{(p)} \equiv \det \|\gamma_{ij}\|. \quad (38)$$

Taking into account (36)–(37), (9) acquire the following gauge-fixed form (recall definition of the induced metric g_{ab} (4)):

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu} \dot{X}^\nu = 0, \quad g_{0i} = 0, \quad g_{ij} - b_0 \gamma_{ij} = 0, \quad (39)$$

where b_0 is the same constant as in (16).

We can impose gauge-fixing of reparametrization invariance within the Nambu-Goto-type formalism (Sect. 2.2) similar to (36) (using notation (31)):

$$\bar{g}_{0i} = 0 \quad (i = 1, \dots, p), \quad \bar{g}_{00} = -\epsilon \bar{b}_0, \quad \bar{b}_0 = \text{const} > 0. \quad (40)$$

Here the constant \bar{b}_0 is an arbitrary gauge-fixing parameter which now can be identified with the arbitrary integration constant b_0 constant (16) within the Polyakov-type formalism. Also, in analogy with the ansatz (37) within the Polyakov-type formulation we will use the ansatz:

$$\partial_i u + \mathcal{A}_i = 0, \quad (41)$$

which is consistent for spherically symmetric bulk space-time Maxwell fields \mathcal{A}_μ and whose physical meaning is that we choose the lightlike direction in (23) to coincide with the brane proper-time direction on the world-volume.

With (40)–(41) (33) implies (cf. (38) above):

$$\partial_0 g^{(p)} = 0 \quad \text{where} \quad g^{(p)} \equiv \det \|g_{ij}\|, \quad (42)$$

g_{ij} being the spacelike part of the induced metric (4).

Taking into account (40)–(42), the equations of motion (33) and (34) (or, equivalently, (35)) reduce to (cf. (39)–(14)):

$$g_{00} \equiv \dot{X}^\mu G_{\mu\nu} \dot{X}^\nu = 0, \quad g_{0i} = 0, \quad T^2 = \frac{1}{\bar{b}_0} (\partial_0 u + \mathcal{A}_0)^2 \quad (\text{i.e.} \quad \partial_i T = 0). \quad (43)$$

3.2 Spherically symmetric backgrounds

Here we will be interested in static spherically symmetric solutions of Einstein-Maxwell equations (see (56)–(57) below). We will consider the following generic form of static spherically symmetric metric:

$$ds^2 = -A(\eta)dt^2 + \frac{d\eta^2}{A(\eta)} + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j, \tag{44}$$

or, in Eddington-Finkelstein coordinates [55,56] ($dt = dv - \frac{d\eta}{A(\eta)}$):

$$ds^2 = -A(\eta)dv^2 + 2dv d\eta + C(\eta)h_{ij}(\theta)d\theta^i d\theta^j. \tag{45}$$

Here h_{ij} indicates the standard metric on the sphere S^p . The radial-like coordinate η will vary in general from $-\infty$ to $+\infty$.

We will consider the simplest ansatz for the *LL-brane* embedding coordinates:

$$X^0 \equiv v = \tau, \quad X^1 \equiv \eta = \eta(\tau), \quad X^i \equiv \theta^i = \sigma^i \quad (i = 1, \dots, p) \tag{46}$$

Now, the *LL-brane* (39) together with (38) yield:

$$-A(\eta) + 2\dot{\eta} = 0, \quad \partial_\tau C = \dot{\eta} \partial_\eta C \Big|_{\eta=\eta(\tau)} = 0. \tag{47}$$

First, we will consider the case of $C(\eta)$ as non-trivial function of η (i.e., proper spherically symmetric space-time). In this case (47) imply:

$$\dot{\eta} = 0 \rightarrow \eta(\tau) = \eta_0 = \text{const}, \quad A(\eta_0) = 0. \tag{48}$$

Equation (48) tells us that consistency of *LL-brane* dynamics in a proper spherically symmetric gravitational background of codimension one requires the latter to possess a horizon (at some $\eta = \eta_0$), which is automatically occupied by the *LL-brane* (“horizon straddling” according to the terminology of Ref. [5]). Similar property—“horizon straddling”, has been found also for *LL-branes* moving in rotating axially symmetric (Kerr or Kerr-Newman) and rotating cylindrically symmetric black hole backgrounds [19–21].

With the embedding ansatz (46) and assuming the bulk Maxwell field to be purely electric static one ($\mathcal{F}_{0\eta} = \mathcal{F}_{v\eta} \neq 0$, the rest being zero; this is the relevant case to be discussed in what follows), (27) yields the simple relation: $\partial_i \chi = 0$, i.e., $\chi = \chi(\tau)$. Further, the only non-trivial contribution of the second order (w.r.t. world-volume proper time derivative) X^μ -equations of motion (28) arises for $\mu = v$, where the latter takes the form of an evolution equation for the dynamical tension $\chi(\tau)$:

$$\partial_\tau \chi + \chi \frac{1}{2} \left[\partial_\eta A + pb_0 \partial_\eta \ln C \right]_{\eta=\eta_0} - q \sqrt{p! F^2 \mathcal{F}_{v\eta}} + \beta b_0^{p/2} \mathcal{F} = 0 \tag{49}$$

(recall (10) and the definition (29) of the Kalb-Ramond field-strength \mathcal{F}). In the case of absence of couplings to bulk space-time gauge fields, (49) yields exponential “inflation”/“deflation” at large times for the dynamical *LL-brane* tension:

$$\chi(\tau) = \chi_0 \exp \left\{ -\tau \frac{1}{2} (\partial_\eta A + p b_0 \partial_\eta \ln C)_{\eta=\eta_0} \right\}, \quad \chi_0 = \text{const.} \tag{50}$$

Similarly to the “horizon straddling” property, exponential “inflation”/“deflation” for the *LL-brane* tension has also been found in the case of test *LL-brane* motion in rotating axially symmetric and rotating cylindrically symmetric black hole backgrounds (for details we refer to Refs. [17–21]). This phenomenon is an analog of the “mass inflation” effect around black hole horizons [23,24].

3.3 Product-type gravitational backgrounds: Bertotti-Robinson space-time

Consider now the case $C(\eta) = \text{const}$ in (45), i.e., the corresponding space-time manifold is of product type $\Sigma_2 \times S^p$. A physically relevant example is the Bertotti-Robinson [39,40,42] space-time in $D = 4$ (i.e., $p = 2$) with (non-extremal) metric in standard coordinates $(\tilde{t}, \zeta, \theta^i)$ (cf.[42]):

$$ds^2 = r_0^2 \left[-\sinh^2 \zeta d\tilde{t}^2 + d\zeta^2 + d\theta^2 + \sin^2 \theta d\varphi^2 \right] \tag{51}$$

describing Anti-de-Sitter₂ \times S^2 . Upon coordinate change $(\tilde{t}, \zeta) \rightarrow (t, \eta)$:

$$t = e^{\tilde{t}} \coth \zeta, \quad \eta = e^{-\tilde{t}} \sinh \zeta \tag{52}$$

the metric (51) acquires the form:

$$ds^2 = r_0^2 \left[-\eta^2 dt^2 + \frac{d\eta^2}{\eta^2} + d\theta^2 + \sin^2 \theta d\varphi^2 \right], \tag{53}$$

or in Eddington-Finkelstein (EF) form $\left(dt = \frac{1}{r_0} dv - \frac{d\eta}{\eta^2} \right)$:

$$ds^2 = -\frac{\eta^2}{r_0^2} dv^2 + 2dv d\eta + r_0^2 \left[d\theta^2 + \sin^2 \theta d\varphi^2 \right]. \tag{54}$$

At $\eta = 0$ the Bertotti-Robinson metric (53) (or (54)) possesses a horizon. Further, we will consider the case of Bertotti-Robinson universe with constant electric field $\mathcal{F}_{v\eta} = \pm \frac{1}{2r_0\sqrt{\pi}}$. In the present case the second (47) is trivially satisfied whereas the first one yields: $\eta(\tau) = \eta(0) \left(1 - \tau \frac{\eta(0)}{2r_0^2} \right)^{-1}$. In particular, if the *LL-brane* is initially (at $\tau = 0$) located on the Bertotti-Robinson horizon $\eta = 0$, it will stay there permanently.

4 Lagrangian formulation of bulk gravity/gauge-field system interacting with lightlike branes

Let us now consider self-consistent bulk Einstein-Maxwell-Kalb-Ramond system coupled to *two* distantly separated charged codimension-one *lightlike* p -branes (in this case $D = (p + 1) + 1$). It is described by the following action:

$$S = \int d^D x \sqrt{-G} \left[\frac{R(G)}{16\pi} - \frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} - \frac{1}{D!2} \mathcal{F}_{\mu_1 \dots \mu_D} \mathcal{F}^{\mu_1 \dots \mu_D} \right] + \tilde{S}_{\text{Pol}}[q, \beta] + \tilde{S}_{\text{NG}}[\bar{q}, \bar{\beta}]. \tag{55}$$

Here again $\mathcal{F}_{\mu\nu}$ and $\mathcal{F}_{\mu_1 \dots \mu_D}$ are the Maxwell and Kalb-Ramond field-strengths (29). The last two terms on the r.h.s. of (55) denote the reparametrization invariant world-volume actions of the two *LL-branes* coupled to the bulk space-time gauge fields—the first one in the Polyakov-type form (26) and the second one in the Nambu-Goto-type form (30) (below we will need to choose the sign $\epsilon = -1$ in the latter). The objects pertaining to the second *LL-brane* will be denoted with bars.

The corresponding Einstein-Maxwell-Kalb-Ramond equations of motion derived from the action (55) read:

$$R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R = 8\pi \left(T_{\mu\nu}^{(EM)} + T_{\mu\nu}^{(KR)} + T_{\mu\nu}^{(brane)} + \bar{T}_{\mu\nu}^{(brane)} \right), \tag{56}$$

$$\partial_\nu \left(\sqrt{-G} \mathcal{F}^{\mu\nu} \right) + q \int d^{p+1} \sigma \delta^{(D)}(x - X(\sigma)) \epsilon^{ab_1 \dots b_p} F_{b_1 \dots b_p} \partial_a X^\mu - \epsilon \bar{q} \int d^{p+1} \sigma \delta^{(D)}(x - \bar{X}(\sigma)) \sqrt{|\bar{g}|} \bar{g}^{ab} \partial_a \bar{X}^\mu \frac{\partial_{b\mu} u + \bar{q} A_b}{T} = 0, \tag{57}$$

$$\epsilon^{\nu\mu_1 \dots \mu_{p+1}} \partial_\nu \mathcal{F} - \beta \int d^{p+1} \sigma \delta^{(D)}(x - X(\sigma)) \epsilon^{a_1 \dots a_{p+1}} \partial_{a_1} X^{\mu_1} \dots \partial_{a_{p+1}} X^{\mu_{p+1}} - \bar{\beta} \int d^{p+1} \sigma \delta^{(D)}(x - \bar{X}(\sigma)) \epsilon^{a_1 \dots a_{p+1}} \partial_{a_1} \bar{X}^{\mu_1} \dots \partial_{a_{p+1}} \bar{X}^{\mu_{p+1}} = 0. \tag{58}$$

Here $X(\sigma)$ and $\bar{X}(\sigma)$ denoted the space-time embeddings of the two *LL-branes*, \bar{g}_{ab} is as defined in (31), and in (58) we have used the last relation (29). The explicit form of the energy-momentum tensors read:

$$T_{\mu\nu}^{(EM)} = \mathcal{F}_{\mu\kappa} \mathcal{F}_{\nu\lambda} G^{\kappa\lambda} - G_{\mu\nu} \frac{1}{4} \mathcal{F}_{\rho\kappa} \mathcal{F}_{\sigma\lambda} G^{\rho\sigma} G^{\kappa\lambda}, \tag{59}$$

$$T_{\mu\nu}^{(KR)} = \frac{1}{(D-1)!} \left[\mathcal{F}_{\mu\lambda_1 \dots \lambda_{D-1}} \mathcal{F}_{\nu}{}^{\lambda_1 \dots \lambda_{D-1}} - \frac{1}{2D} G_{\mu\nu} \mathcal{F}_{\lambda_1 \dots \lambda_D} \mathcal{F}^{\lambda_1 \dots \lambda_D} \right] = -\frac{1}{2} \mathcal{F}^2 G_{\mu\nu}, \tag{60}$$

$$T_{(brane)}^{\mu\nu} = - \int d^{p+1} \sigma \frac{\delta^{(D)}(x - X(\sigma))}{\sqrt{-G}} \chi \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu, \tag{61}$$

$$\bar{T}_{(brane)}^{\mu\nu} = - \int d^{p+1} \sigma \frac{\delta^{(D)}(x - \bar{X}(\sigma))}{\sqrt{-G}} T \sqrt{|\bar{g}|} \bar{g}^{ab} \partial_a \bar{X}^\mu \partial_b \bar{X}^\nu. \tag{62}$$

The *LL-brane* stress-energy tensors (61)–(62) are straightforwardly derived upon varying w.r.t. $G^{\mu\nu}$ from the world-volume actions (26) and (30), respectively (recall $\chi \equiv \frac{\Phi}{\sqrt{-\gamma}}$ is the variable brane tension as in (7)).

The equations of motion for the *LL-branes* have already been written down in Sect. 2—(8)–(9) and (27)–(27) for the first *LL-brane* (in the Polyakov-type formulation) and (32)–(34) for the second *LL-brane* (in the Nambu-Goto-type formulation).

In what follows we will employ the embedding ansatz (46) for both *LL-branes* together with (48) as well as the world-volume reparametrization gauge-fixings (36) and (40). In this case the Kalb-Ramond equations of motion (58) reduce to:

$$\partial_\eta \mathcal{F} + \beta \delta(\eta - \eta_0) + \bar{\beta} \delta(\eta - \bar{\eta}_0) = 0 \tag{63}$$

implying:

$$\mathcal{F} \Big|_{\eta \rightarrow \eta_0 - 0} - \mathcal{F} \Big|_{\eta \rightarrow \eta_0 + 0} = \beta, \quad \mathcal{F} \Big|_{\eta \rightarrow \bar{\eta}_0 - 0} - \mathcal{F} \Big|_{\eta \rightarrow \bar{\eta}_0 + 0} = \bar{\beta} \tag{64}$$

with $\mathcal{F} \Big|_{\eta \rightarrow \eta_0 + 0} = \mathcal{F} \Big|_{\eta \rightarrow \bar{\eta}_0 - 0}$. Here η_0 and $\bar{\eta}_0$ indicate the positions of both *LL-branes* which according to (48) must represent horizons w.r.t. the embedding space-time geometry. We will assume $\eta_0 < \bar{\eta}_0$ (see below). Thus, according to (63)–(64) a piece-wise varying non-negative cosmological constant $\Lambda = 4\pi \mathcal{F}^2$ (cf. (60)) is dynamically generated in the various space-time regions w.r.t. the horizons at $\eta = \eta_0$ and $\eta = \bar{\eta}_0$, respectively.

Next, we consider static spherically symmetric solution of Maxwell equations (57) which in this case, using ansatz (37) and the gauge-fixing conditions (40) together with *LL-brane* embeddings (46), acquire the form:

$$\partial_\eta \left(C^{p/2}(\eta) \mathcal{F}_{v\eta}(\eta) \right) - q \frac{\sqrt{p! F^2}}{b_0^{p/2}} C^{p/2}(\eta_0) \delta(\eta - \eta_0) - \bar{q} C^{p/2}(\bar{\eta}_0) \delta(\eta - \bar{\eta}_0) = 0. \tag{65}$$

As mentioned above in Sect. 3, the evolution (49) for the dynamical *LL-brane* tension χ remains the only non-trivial contribution of the second order X^μ -equations of motion (28) for $\mu = v$ within the Polyakov-type formulation when employing the embedding ansatz (46). In the present context of looking for self-consistent solution of the bulk gravity/gauge-field system coupled to the two *LL-branes* (55) the “force” terms in (49) (the geodesic ones containing the Christoffel connection coefficients as well as those coming from the *LL-brane* coupling to the bulk Maxwell and Kalb-Ramond gauge fields) contain discontinuities across the horizon occupied by the corresponding *LL-brane*. The discontinuity problem is resolved following the approach in Refs. [3, 4] (see also the regularization approach in Ref. [57], Appendix A) by taking mean values of the “force” terms across the discontinuity at $\eta = \eta_0$. Thus,

(49) is replaced by:

$$\begin{aligned} \partial_\tau \chi + \chi \frac{1}{2} \left(\langle \partial_\eta A \rangle_{\eta=\eta_0} + pb_0 \langle \partial_\eta \ln C \rangle_{\eta=\eta_0} \right) - q \sqrt{p!F^2} \langle \mathcal{F}_{v\eta} \rangle_{\eta=\eta_0} \\ + \beta b_0^{p/2} \langle \mathcal{F} \rangle_{\eta=\eta_0} = 0 \end{aligned} \tag{66}$$

Here and below we will use the following short-hand notations for ‘‘mean value’’ and ‘‘discontinuity’’ for any relevant quantity across $\eta = \eta_0$:

$$\langle Y \rangle_{\eta=\eta_0} \equiv \frac{1}{2} \left(Y \Big|_{\eta \rightarrow \eta_0+0} + Y \Big|_{\eta \rightarrow \eta_0-0} \right), \quad [Y]_{\eta=\eta_0} \equiv Y \Big|_{\eta \rightarrow \eta_0+0} - Y \Big|_{\eta \rightarrow \eta_0-0}, \tag{67}$$

and similarly across $\eta = \bar{\eta}_0$.

Accordingly, the X^μ -equation for $\mu = v$ within the dual Nambu-Goto-type formulation (32) taking into account the gauge-fixing (40) becomes evolution equation for the dynamical *LL-brane* tension T which we will write for convenience in terms of the rescaled quantity:

$$\bar{\chi} \equiv T \bar{b}_0^{\frac{p-1}{2}} \tag{68}$$

(cf. second relation in (24)):

$$\begin{aligned} \partial_\tau \bar{\chi} + \bar{\chi} \frac{1}{2} \left(\langle \partial_\eta A \rangle_{\eta=\bar{\eta}_0} + p\bar{b}_0 \langle \partial_\eta \ln C \rangle_{\eta=\bar{\eta}_0} \right) - \epsilon \bar{b}_0^{p/2} \bar{q} \langle \mathcal{F}_{v\eta} \rangle_{\eta=\bar{\eta}_0} \\ + \epsilon \bar{b}_0^{p/2} \bar{\beta} \langle \mathcal{F} \rangle_{\eta=\bar{\eta}_0} = 0 \end{aligned} \tag{69}$$

Taking into account the embedding ansatz (46) for both *LL-branes* together with (48), (36) and (40), the *LL-brane* energy-momentum tensors (61)–(62) on the r.h.s. of the Einstein equations of motion (56), which were derived from the underlying world-volume *LL-brane* actions (7) and (30), acquire the form:

$$T_{(brane)}^{\mu\nu} = S^{\mu\nu} \delta(\eta - \eta_0), \quad \bar{T}_{(brane)}^{\mu\nu} = \bar{S}^{\mu\nu} \delta(\eta - \bar{\eta}_0) \tag{70}$$

with surface energy-momentum tensors:

$$S^{\mu\nu} \equiv \frac{\chi}{b_0^{p/2}} \left(\partial_\tau X^\mu \partial_\tau X^\nu - b_0 G^{ij} \partial_i X^\mu \partial_j X^\nu \right)_{v=\tau, \eta=\eta_0, \theta^i=\sigma^i}, \tag{71}$$

$$\bar{S}^{\mu\nu} \equiv \frac{\bar{\chi}}{\epsilon \bar{b}_0^{p/2}} \left(\partial_\tau \bar{X}^\mu \partial_\tau \bar{X}^\nu - \epsilon \bar{b}_0 G^{ij} \partial_i \bar{X}^\mu \partial_j \bar{X}^\nu \right)_{v=\tau, \eta=\bar{\eta}_0, \theta^i=\sigma^i}. \tag{72}$$

Here and below $G_{ij} = C(\eta)h_{ij}(\theta)$ (cf. (45)) and $\bar{\chi}$ is as in (68). For the non-zero components of (71)–(72) (with lower indices) and its trace we find:

$$S_{\eta\eta} = \frac{\chi}{b_0^{p/2}}, \quad S_{ij} = -\frac{\chi}{b_0^{p/2-1}}G_{ij}, \quad S_\lambda^\lambda = -\frac{p\chi}{b_0^{p/2-1}} \tag{73}$$

$$\bar{S}_{\eta\eta} = \epsilon \frac{\bar{\chi}}{\bar{b}_0^{p/2}}, \quad \bar{S}_{ij} = -\frac{\bar{\chi}}{\bar{b}_0^{p/2-1}}G_{ij}, \quad \bar{S}_\lambda^\lambda = -\frac{p\bar{\chi}}{\bar{b}_0^{p/2-1}} \tag{74}$$

Because of the delta-function nature of the *LL-brane* stress-energy tensors (70), the solution of the Einstein-Maxwell-Kalb-Rammond equations (56)–(58) consists of finding “electrovacuum” solutions in each of the three space-time regions ($\eta < \eta_0$), ($\eta_0 < \eta < \bar{\eta}_0$) and ($\eta > \bar{\eta}_0$) and subsequently matching them along the lightlike hypersurfaces ($\eta = \eta_0$) and ($\eta = \bar{\eta}_0$) which themselves must be their pairwise common horizons and which due to (48) will be automatically occupied by the two *LL-branes* (“horizon straddling”).

The systematic formalism for matching different bulk space-time geometries on codimension-one timelike hypersurfaces (“thin shells”) was developed originally in Ref. [3,4] and later generalized in Ref. [5] to the case of lightlike hypersurfaces (“null thin shells”) (for a systematic introduction, see the textbook [58]). In the present case we are interested in static spherically symmetric solutions with metric of the form (45), therefore, due to the simple geometry one can straightforwardly isolate the terms from the Ricci tensor on the l.h.s. of Einstein equations (56) which may yield delta-function contributions ($\sim \delta(\eta - \eta_0)$ and $\sim \delta(\eta - \bar{\eta}_0)$) to be matched with the components of the *LL-brane* surface stress-energy tensors (70)–(72). The metric (45) is continuous at both lightlike hypersurfaces ($\eta = \eta_0$) and ($\eta = \bar{\eta}_0$), but its first derivative w.r.t. η (the normal coordinate w.r.t. both horizons) might exhibit discontinuity across $\eta = \eta_0$ and $\eta = \bar{\eta}_0$. Thus, the terms contributing to delta-function singularities in $R_{\mu\nu}$ are those containing second derivatives w.r.t. η . Separating explicitly the latter we can rewrite (56) in the following form:

$$\begin{aligned} R_{\mu\nu} &\equiv \partial_\eta \Gamma_{\mu\nu}^\eta - \partial_\mu \partial_\nu \ln \sqrt{-G} + \text{non - singular terms} \\ &= 8\pi \left(S_{\mu\nu} - \frac{1}{p} G_{\mu\nu} S_\lambda^\lambda \right) \delta(\eta - \eta_0) \\ &\quad + 8\pi \left(\bar{S}_{\mu\nu} - \frac{1}{p} G_{\mu\nu} \bar{S}_\lambda^\lambda \right) \delta(\eta - \bar{\eta}_0) + \text{non - singular terms.} \end{aligned} \tag{75}$$

For the sake of simplicity we will consider in what follows the case of $D = 4$ -dimensional bulk space-time and, correspondingly, $p = 2$ for the *LL-branes*. The generalization to arbitrary D is straightforward. For further simplification of the numerical constant factors we will choose the following specific (“wrong-sign” Maxwell) form for the Lagrangian of the auxiliary non-dynamical world-volume gauge field in the Polyakov-type *LL-brane* formulation (cf. (5)–(6)):

$$L(F^2) = \frac{1}{4}F^2 \quad \rightarrow \quad b_0 = 2M, \tag{76}$$

where again b_0 is the constant defined in (16) and M denotes the original integration constant in (8).

Substituting in Einstein equations (75) the explicit form of the metric (45) and the *LL-brane* stress-energy expressions (73)–(74) yields the following matching relations at the delta-function singularities (using notations (67) and (68)):

$$[\partial_\eta A]_{\eta_0} = -16\pi \chi, \quad [\partial_\eta A]_{\bar{\eta}_0} = -16\pi \bar{\chi} \tag{77}$$

$$[\partial_\eta \ln C]_{\eta_0} = -\frac{8\pi}{b_0} \chi, \quad [\partial_\eta \ln C]_{\bar{\eta}_0} = -\epsilon \frac{8\pi}{b_0} \bar{\chi} \tag{78}$$

Consistency of (77)–(78) requires that the dynamical tensions $\chi, \bar{\chi}$ of both *LL-branes* must be time-independent, i.e., $\partial_\tau \chi = 0$ and $\partial_\tau \bar{\chi} = 0$ in (66) and (69), respectively.

From the Maxwell equations (65) we obtain the following matchings for the static spherically symmetric electromagnetic field-strength across the *LL-brane* world-volume hypersurfaces (using again notations (67)):

$$[\mathcal{F}_{v\eta}]_{\eta_0} = \frac{2}{\sqrt{b_0}} q, \quad [\mathcal{F}_{v\eta}]_{\bar{\eta}_0} = \bar{q} \tag{79}$$

5 Two-throat wormhole-like solution via lightlike branes

We will seek self-consistent solution of the equations of motion of the coupled Einstein-Maxwell-Kalb-Ramond-*LL-brane* system (Sect. 4) describing an asymmetric wormhole-like space-time with spherically symmetric geometry and two “throats”. The general form of asymmetric wormhole-like metric (in Eddington-Finkelstein coordinates) (45) specifically reads (now $D = 4, p = 2$):

$$ds^2 = -A(\eta)dv^2 + 2dv d\eta + C(\eta) \left[d\theta^2 + \sin^2 \theta d\varphi^2 \right], \tag{80}$$

$$A(\eta_0) = 0, \quad A(\bar{\eta}_0) = 0, \quad A(\eta) > 0 \quad \text{for all } \eta \neq \eta_0, \bar{\eta}_0, \quad \bar{\eta}_0 > \eta_0. \tag{81}$$

The present wormhole-like solution describes *three* pairwise matched space-time regions:

- (i) “left” Bertotti-Robinson “universe” (cf. (54)) where:

$$A(\eta) = \frac{\eta^2}{r_0^2}, \quad C(\eta) = r_0^2, \quad \mathcal{F}_{v\eta} = \epsilon_L \frac{1}{2\sqrt{\pi} r_0} \quad \text{for } \eta < 0, \quad \epsilon_L = \pm 1, \tag{82}$$

with the choice $\eta_0 = 0$ for simplicity;

(ii) “middle” Reissner-Nordström-de-Sitter “universe” with:

$$A(\eta) \equiv A_{RN}(r_0 + \eta) = 1 - \frac{2m}{r_0 + \eta} + \frac{Q^2}{(r_0 + \eta)^2} - \frac{4\pi\beta^2}{3}(r_0 + \eta)^2, \quad (83)$$

$$C(\eta) = (r_0 + \eta)^2, \quad \mathcal{F}_{v\eta} \equiv \mathcal{F}_{vr} \Big|_{RN} = \frac{Q}{\sqrt{4\pi}(r_0 + \eta)^2}$$

for $0 < \eta < \bar{\eta}_0 \equiv \bar{r}_0 - r_0$,

$$A(0) \equiv A_{RN}(r_0) = 0, \quad \partial_\eta A \Big|_{\eta \rightarrow +0} \equiv \partial_r A_{RN} \Big|_{r=r_0} > 0, \quad (85)$$

$$A(\bar{\eta}_0) \equiv A_{RN}(\bar{r}_0) = 0, \quad \partial_\eta A \Big|_{\eta \rightarrow \bar{\eta}_0 - 0} \equiv \partial_r A_{RN} \Big|_{r=\bar{r}_0} < 0, \quad (86)$$

where r_0 and \bar{r}_0 ($\bar{r}_0 > r_0$) are the intermediate (outer Reissner-Nordström) and the outmost (de-Sitter) horizons of the standard Reissner-Nordström-de-Sitter space-time, respectively;

(iii) another “right” Bertotti-Robinson “universe” (cf. (54)) with:

$$A(\eta) = \frac{(\eta - \bar{r}_0 + r_0)^2}{\bar{r}_0^2}, \quad C(\eta) = \bar{r}_0^2$$

$$\mathcal{F}_{v\eta} = \varepsilon_R \frac{1}{2\sqrt{\pi} \bar{r}_0} \quad \text{for } \eta > \bar{\eta}_0 \equiv \bar{r}_0 - r_0, \quad \varepsilon_R = \pm 1. \quad (87)$$

Let us stress that the cosmological constant:

$$\Lambda = 4\pi\beta^2 \quad (88)$$

in the middle Reissner-Nordström-de-Sitter space-time region (ii) is dynamically generated by the presence of the *LL-branes* according to (63)–(64), which in particular implies $\beta = -\bar{\beta}$.

$\mathcal{F}_{v\eta}$'s are the respective Maxwell field-strengths and the Reissner-Nordström charge parameter Q is determined from the discontinuities of $\mathcal{F}_{v\eta}$ in Maxwell equations (65) across each of the two charged *LL-branes*:

$$Q = r_0 \left[\varepsilon_L + \sqrt{\frac{\pi}{b_0}} 4qr_0 \right] \quad \text{and} \quad Q = \bar{r}_0 \left[\varepsilon_R - \sqrt{4\pi} \bar{q} \bar{r}_0 \right]. \quad (89)$$

In (83)–(84) we have used the standard coordinate notations for the Reissner-Nordström-de-Sitter metric coefficients and Coulomb field strength:

$$A_{RN}(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2, \quad \mathcal{F}_{vr} \Big|_{RN} = \mathcal{F}_{0r} \Big|_{RN} = \frac{Q}{\sqrt{4\pi}r^2}. \quad (90)$$

It now remains to substitute the expressions (82)–(87) into the set of (77)–(78) (resulting from the Einstein equations (75)) and (79) (resulting from the Maxwell equations (65)) and the equations for the dynamical brane tensions (66) and (69) (with

vanishing time-derivative terms, cf. (77)–(78) above). This will yield a set of algebraic relations determining all parameters of the “two-throat” wormhole-like solution (82)–(87) as functions of the three free parameters $\beta (= -\bar{\beta})$, q , \bar{q} —the coupling constants of the two *LL-branes*.

First, (78) yield:

$$r_0 = \frac{b_0}{4\pi|\chi|}, \quad \bar{r}_0 = \frac{\bar{b}_0}{4\pi|\bar{\chi}|}, \tag{91}$$

where consistency requires:

$$\chi < 0, \quad \bar{\chi} < 0, \quad \epsilon = -1. \tag{92}$$

Inserting expressions (91) into (85)–(86) allows to express the mass parameter m of the “middle” Reissner-Nordström-de-Sitter “universe” in two ways:

$$m = \frac{b_0}{4\pi|\chi|} \left(1 - 2b_0 - \frac{\beta^2 b_0^2}{6\pi\chi^2} \right), \quad m = \frac{\bar{b}_0}{4\pi|\bar{\chi}|} \left(1 + 2\bar{b}_0 - \frac{\beta^2 \bar{b}_0^2}{6\pi\bar{\chi}^2} \right) \tag{93}$$

Further, (91) together with (77), (66), (69) and (79) imply quadratic equations for the *LL-brane* tensions:

$$\chi^2 + \frac{\epsilon_L q}{\sqrt{4\pi b_0}} |\chi| + \frac{1}{16\pi} (b_0 \beta^2 + 4q^2) = 0, \tag{94}$$

$$\bar{\chi}^2 + \frac{\epsilon_R \bar{q}}{4\sqrt{\pi}} |\bar{\chi}| - \frac{\bar{b}_0}{16\pi} (\beta^2 + \bar{q}^2) = 0, \tag{95}$$

In what follows it is more convenient to rescale the charge parameter of the first *LL-brane*:

$$q \rightarrow \hat{q} \equiv \frac{2}{\sqrt{b_0}} q \tag{96}$$

so that \hat{q} has the same physical meaning of *LL-brane* surface electric charge density as \bar{q} (cf. (65) and (79)). With the notation (96) (94) acquires a form completely analogous to (95):

$$\chi^2 + \frac{\epsilon_L \hat{q}}{4\sqrt{\pi}} |\chi| + \frac{b_0}{16\pi} (\beta^2 + \hat{q}^2) = 0. \tag{97}$$

Existence of solutions to (97) (or (94)) requires the following condition on the signs of the constant electric field of the “left” Bertotti-Robinson “universe” (82) and the charge of the “left” *LL-brane*:

$$\epsilon_L \text{sign}(\hat{q}) = -1. \tag{98}$$

Thus, the solutions of (97) and (95) read (using notation (96)):

$$|\chi| = \frac{|\widehat{q}|}{8\sqrt{\pi}} \left[1 \pm \sqrt{1 - 4b_0(1 + \beta^2/\widehat{q}^2)} \right], \quad (99)$$

$$|\bar{\chi}| = \frac{|\bar{q}|}{8\sqrt{\pi}} \left[-\varepsilon_R \operatorname{sign}(\bar{q}) + \sqrt{1 + 4\bar{b}_0(1 + \beta^2/\bar{q}^2)} \right]. \quad (100)$$

The expression (99) implies a constraint on the parameter b_0 :

$$b_0 < 1/4(1 + \beta^2/\widehat{q}^2). \quad (101)$$

Finally, we need to insert the expressions (91) and (99)–(100) into (89) and (93) which yield accordingly (taking into account (98)):

$$r_0 \varepsilon_L [1 - 2\sqrt{\pi}|\widehat{q}|r_0] = \bar{r}_0 \varepsilon_R [1 - 2\sqrt{\pi}\varepsilon_R \bar{q}\bar{r}_0], \quad (102)$$

$$r_0 \left[1 - 2\sqrt{\pi}|\widehat{q}|r_0 + 2\pi(\widehat{q}^2 - \frac{\beta^2}{3})r_0^2 \right] = \bar{r}_0 \left[1 - 2\sqrt{\pi}\varepsilon_R \bar{q}\bar{r}_0 + 2\pi(\bar{q}^2 - \frac{\beta^2}{3})\bar{r}_0^2 \right]. \quad (103)$$

The consistent solutions of (102)–(103) read:

$$\bar{r}_0 = \alpha r_0, \quad r_0 = \frac{\alpha - 1}{2\sqrt{\pi}(\alpha^2|\bar{q}| - |\widehat{q}|)}, \quad (104)$$

where:

$$\alpha \equiv \left(\frac{\widehat{q}^2 - \beta^2/3}{\bar{q}^2 - \beta^2/3} \right)^{1/3} > 1, \quad \alpha^2 > \frac{|\widehat{q}|}{|\bar{q}|} \quad (105)$$

together with the following conditions on the sign factors—either:

$$\widehat{q} < 0, \quad \bar{q} > 0, \quad \varepsilon_L = \varepsilon_R = +1 \quad (106)$$

or:

$$\widehat{q} > 0, \quad \bar{q} < 0, \quad \varepsilon_L = \varepsilon_R = -1 \quad (107)$$

i.e., the electric charges of both *LL-branes* must be opposite w.r.t. each other and the constant electric fields in the “left” and “right” Bertotti-Robinson “universes” must have equal signs.

Inequalities (105) and (101) imply explicitly the following allowed ranges of the free parameters $(\beta, \widehat{q}, \bar{q})$ of the “two-throat” wormhole-like solution (82)–(87):

$$\text{either } \beta^2 < 3\bar{q}^2, \quad |\widehat{q}| > |\bar{q}| \quad \text{or} \quad \beta^2 > 3\bar{q}^2, \quad |\widehat{q}| < |\bar{q}|, \quad (108)$$

and (using the short-hand notation α from (105)):

$$\frac{(\alpha - 1)|\widehat{q}|(1 + \beta^2/\widehat{q}^2)}{\alpha^2|\bar{q}| - |\widehat{q}|} < 2. \quad (109)$$

Inequalities (108)–(109) imply upper and lower bounds on the dynamically generated (due to the *LL-branes*) cosmological constant $\Lambda = 4\pi\beta^2$ (88) in the middle Reissner-Nordström-de-Sitter space-time region (83)–(86). In particular, we find that Λ cannot be arbitrary small.

Let us also note that for a special fine-tuning of the *LL-brane* parameters $\alpha = |\widehat{q}|/|\bar{q}|$ it follows from (89) and (104) that $Q = 0$, i.e., the electric field inside the “middle” universe (83)–(86) vanishes and the latter becomes an electrically neutral Schwarzschild-de-Sitter region.

6 Discussions and conclusions

In the present work we have explored the use of (codimension-one) *LL-branes* for construction of new asymmetric wormhole-like solutions of Einstein-Maxwell-Kalb-Ramond system self-consistently coupled to two widely separated *LL-branes* which describe a sequence of spontaneous compactification/decompactification transitions of space-time. In the course of work we have strongly emphasized the crucial properties of the dynamics of *LL-branes* interacting with gravity and bulk space-time gauge fields:

- (i) “Horizon straddling”—automatic positioning of *LL-branes* on (one of) the horizon(s) of the bulk space-time geometry;
- (ii) Intrinsic nature of the *LL-brane* tension as an additional *dynamical degree of freedom* unlike the case of standard Nambu-Goto *p-branes*; (where it is a given *ad hoc* constant), and which might in particular acquire negative values;
- (iii) The stress-energy tensors of the *LL-branes* are systematically derived from the underlying *LL-brane* Lagrangian actions and provide the appropriate source terms on the r.h.s. of Einstein equations to enable the existence of consistent non-trivial wormhole-like solutions;
- (iv) Electrically charged *LL-branes* naturally produce *asymmetric* wormholes with the *LL-branes* themselves materializing the wormhole “throats” and uniquely determining the pertinent wormhole parameters.
- (v) *LL-branes* naturally couple to 3-index Kalb-Ramond bulk space-time gauge fields which results in *dynamical* generation of space-time varying cosmological constant.

Let us point out that the above asymmetric “two-throat” wormhole, connecting through the first “throat” a “left” compactified Bertotti-Robinson universe to a “middle” decompactified Reissner-Nordström space-time region and subsequently connecting the latter through the second “throat” to another “right” compactified Bertotti-Robinson universe, is *traversable* w.r.t. the *proper time* of a traveling observer. This property is similar to the *proper time* traversability of other symmetric and asymmetric

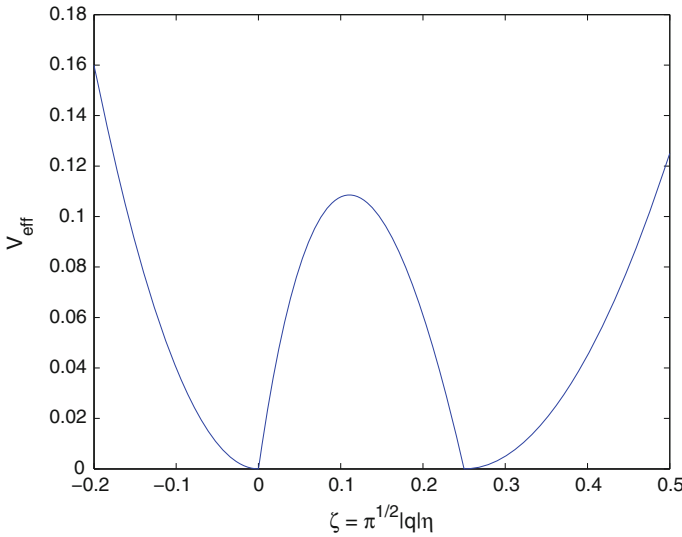


Fig. 1 Shape of $\mathcal{V}_{\text{eff}}(\eta)$ as a function of the rescaled coordinate $\zeta = \sqrt{\pi}|q|\eta$

(“one-throat”) wormholes produced by *LL-brane* sitting on the wormhole “throat” [19–21, 25, 26, 38].

Indeed, let us consider test particle (“traveling observer”) dynamics in the asymmetric wormhole background given by (82)–(87), which is described by the action:

$$S_{\text{particle}} = \frac{1}{2} \int d\lambda \left[\frac{1}{e} \dot{x}^\mu \dot{x}^\nu G_{\mu\nu} - em_0^2 \right]. \tag{110}$$

Using energy \mathcal{E} and orbital momentum \mathcal{J} conservation and introducing the *proper* world-line time s ($\frac{ds}{d\lambda} = em_0$), the “mass-shell” equation (the equation w.r.t. the “einbein” e produced by the action (110)) yields:

$$\left(\frac{d\eta}{ds} \right)^2 + \mathcal{V}_{\text{eff}}(\eta) = \frac{\mathcal{E}^2}{m_0^2}, \quad \mathcal{V}_{\text{eff}}(\eta) \equiv A(\eta) \left(1 + \frac{\mathcal{J}^2}{m_0^2 C(\eta)} \right) \tag{111}$$

where the metric coefficients $\mathcal{A}(\eta)$, $C(\eta)$ are given in (82)–(87).

The shape of the “effective potential” is depicted on Fig. 1 in the simple case of $\mathcal{J} = 0$, i.e., for a purely “radial” motion along the η -direction. In particular, in both “left” and “right” Bertotti-Robinson universes it has harmonic-oscillator-type form. Therefore, a “radially” moving traveling observer (with sufficiently large energy \mathcal{E}) will periodically “shuttle” along the η -direction within *finite proper-time* intervals between the two turning points $\eta_{(-)} < 0$ in the “left” Bertotti-Robinson universe and $\eta_{(+)} > \bar{r}_0 - r_0$ in the “right” Bertotti-Robinson universe, where $\mathcal{V}_{\text{eff}}(\eta_{(\pm)}) = \mathcal{E}^2/m_0^2$.

On the other hand let us point out that, as in the case of the previously constructed symmetric and asymmetric wormholes via *LL-branes* sitting on their “throats” [19–21, 25, 26, 38], the present “two-throat” wormhole is *not* traversable w.r.t. the “laboratory” time of a static observer in either of the three universes.

Let us also mention the crucial role of *LL-branes* in constructing non-trivial examples of *non-singular* black holes, i.e., solutions of Einstein equations with black hole type geometry in the bulk space-time, in particular possessing horizons, but with *no* space-time singularities in the center of the geometry. For further details we refer to [46], where a solution of the Einstein-Maxwell-Kalb-Ramond system coupled to a charged *LL-brane* has been obtained describing a regular black hole. The space-time manifold of the latter consists of de Sitter interior region and exterior Reissner-Nordström region glued together along their common horizon (it is the *inner* horizon from the Reissner-Nordström side).

Finally, let us point out one possible direction for generalizing the present results to higher-dimensional cases. Notice that one generalization of Bertotti-Robinson solutions in $D = 6$ is the famous Freund-Rubin solution [59] of the form $AdS_4 \times S^2$. Another generalization of the form $Minkowski_4 \times S^2$ was found in [60] upon introducing a bare cosmological constant in $D = 6$. One could envisage a construction of a $D = 6$ -dimensional wormhole-like solution of gravity/gauge-field system self-consistently interacting with two (widely separated) codimension-one lightlike branes describing a transition from a “left” Freund-Rubin-type universe via “intermediate” $D = 6$ Reissner-Nordström-de-Sitter region (cf. [61]) to a different “right” Freund-Rubin-type universe where the lightlike 4-branes occupy the interfaces between the various universes.

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